Spreadsheet-Based Reliability Simulation: Understanding Test and Customer Data Towards Improving Reliability Performance

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Outline

- Brief Review of Reliability Statistics Concepts
- TTF Simulation: Exponential
- Weibull and Lognormal
- Parametric Bootstrap Applications
  - Case I: Accelerated Life Test Modeling
  - Case II: Customer Burn-In Optimization
- Summary and Conclusions
Brief Review of Reliability Statistics Concepts

- **CDF** – Cumulative Distribution Function
  
  $F(t)$ is probability that a unit has failed by time $t$

  $F(0) = 0 < F(t) < F(\infty) = 1$  
  the inverse $t(F)$ is single-valued  
  $F(t + \Delta t) > F(t)$

- **pdf** – probability density function

  $f(t)dt$ is probably that a unit has failed between time $t$ and $t + dt$

  $$f(t) = \frac{dF}{dt}$$

- Failure Rate – also called hazard function

  $h(t) = f(t)/\{1 - F(t)\}$ fractional failure rate at time $t$
• $F(t) = 1 - e^{-t/\tau}$

$f(t) = \tau^{-1}e^{-t/\tau}$

failure rate $h(t) = \text{constant} = 1/\tau$

To draw samples, solve for $t \Rightarrow t = -\tau \ln(1 - F)$

Replace $F$ with $\text{rand}()$ ... random number between 0 and 1

To implement censoring

$= \text{min}(\text{sample, censor time})$

Compare sample average (mean) and stdev to population $MTTF = \tau$

How does this depend on sample size?
Weibull Distribution $F(t) = 1 - e^{-(t/\tau)^\beta}$

Solve for $t$ for parametric bootstrap $t = \tau(-\ln(1 - F))^{1/\beta}$

Where $F = U[0,1] = \text{rand}()$

Lognormal Distribution $pdf$ $f(\ln t) = N(\mu, \sigma)$

Solving to draw a sample: $t = \exp(\sigma \times \text{normsinv}(:\text{rand}()) + \mu)$

*Note: a normal ttf distribution would allow for negative failure times … N/A!*
TTF Simulation: Weibull and Lognormal

- Spreadsheet modifications exercise
- Modify previous spreadsheets to represent
  - Lognormal distribution with
    - μ = σ = 3 ln hrs
    - μ = 3 and σ = 3
  - Weibull distribution with
    - τ = 200hrs and β = -0.5
    - τ = 200hrs and β = 2.0
Case I: Accelerated Life Test Modeling

- Confidence intervals are important in assessing the reliability performance delta between two products/technologies.
- Activation energies should be reported with confidence intervals.
- How many 3-temperature ALT data points would be needed to enable you to report an activation energy of $E_a = 2.5348\text{eV}$ (implying confidence interval of $\pm 0.0001\text{ eV}$)
Simulating Binomial Distribution

• Given an average failure fraction of \( x = 0.2 \), what range of failures would you expect to witness from 200 units? *Excel function* `binom.dist()`

• The inverse question ... from 200 units, 35 are found to fail a reliability test. What is the confidence interval on the population failure fraction. *Excel function* `binom.inv()`
Case II: Customer Burn-In Optimization

• Understand how Burn-In (BI) dpm (die per million) variation depends on sample size for a Weibull infant mortality with $\beta = -0.5$.
  – Create set of Weibull TTFs
    • for $\tau = 200$ hrs and sample sizes of 1000, 10000, 100000 and burn-in time of $\Delta t = 12$ hrs
  – Use inverse binomial distribution from Weibull CDF.
Summary and Conclusions

- All Excel files created during this tutorial can be added to your conference thumb drive.
- Monte Carlo simulation of reliability data can provide a deep understanding of confidence intervals as a function of sample size and distribution type.
- Spreadsheet example covered today easily portable to other platforms (eg Google Sheets)
BACKUP
### Exponential example: cumulative mean funnels

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>population</td>
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<tr>
<td>2</td>
<td>population MTTF, ( \tau ) (hours)</td>
<td>1.00E+06</td>
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<tr>
<td>3</td>
<td>sample mean, ( t_{ave} ) (hours)</td>
<td>4.8E+05</td>
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<tr>
<td>4</td>
<td>draw #, ( i )</td>
<td>( t_{ff} ) (hours)</td>
<td>cumulative ( t_{ave} )</td>
<td>( t_{ff} ) (hours)</td>
<td>cumulative ( t_{ave} )</td>
<td>( t_{ff} ) (hours)</td>
<td>cumulative ( t_{ave} )</td>
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</tr>
<tr>
<td>5</td>
<td>1</td>
<td>4.8E+05</td>
<td>4.8E+05</td>
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</tbody>
</table>

- **$B$4 formula**
  
  \[
  =\text{AVERAGE}(B6:B305) +N(" sample mean won't match population mean in $B$52 !! ")
  \]

- **$B$6 formula**
  
  \[
  =-B52*LN(1-RAND()) +N(" cell B$52 contains the population MTTF ")
  \]

- **$C$6 formula**
  
  \[
  =\text{AVERAGE}(B$6:B6) +N(" the 'S' pins start of averaging range to row 6 ")
  \]

**Chart 2**

\[
=\text{SERIES}('cumulative mean funnels!'B5$6:B5$305,1)
\]

**Chart 1**

\[
=\text{SERIES}('cumulative mean funnels!'C5$6:C5$305,22) etc
\]

- **time to fail per draw #**

  - 600000
  - 600000

- **MTTF confidence funnels with increasing sample size**

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Exponential example: distribution of sample means

=Average(B$11:B111) + N(" we'll correct end of range after filling down ")

=D$52*LN(1-RAND())

=C$Y3*D$52 +N(" the above fraction times the population MTTF in D$52 ")

=D$54 formula

=COUNTIFS($B4:$CW4, ">"&C$Y3, $B4:$CW4, "<="&D$Z3)

Chart 1

### Exponential example: time ordered with censoring

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>number of samples (&lt;=300)</td>
<td>2</td>
<td></td>
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<td></td>
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<tr>
<td>2</td>
<td></td>
<td></td>
<td>censoring time (hours)</td>
<td>200</td>
<td></td>
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<td>3</td>
<td></td>
<td></td>
<td>population MTTF, ( \tau ) (hours)</td>
<td>1000</td>
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<tr>
<td>4</td>
<td></td>
<td></td>
<td>sample mean, ( t_{\text{ave}} ) (hours)</td>
<td>1023.3</td>
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<td></td>
<td>time (hours)</td>
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<tr>
<td>5</td>
<td></td>
<td></td>
<td><strong>unit #</strong> ( t_{f} ) (hours)</td>
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<tr>
<td>6</td>
<td></td>
<td></td>
<td>time-ordered start, censored time</td>
<td></td>
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<td>7</td>
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<td>end, for JMP</td>
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</tbody>
</table>

- **$B$4 formula**
  
  \[ \text{Chart 1} = \text{SERIES}("censored time","time ordered with censoring"!D$6:D$305,2) \]

- **$B$6 formula**
  
  \[ -B$2*LN(1-RAND()) \]

- **$B$7 formula**
  
  \[ \text{IF} ( A7 > $E$1, "", -B$2*LN(1-RAND()) ) \]

- **$C$6 formula**
  
  \[ \text{LARGE}($B$6:$B$305,$E$1+1-$A$6) \]

- **$C$7 formula**
  
  \[ \text{IF} (B7<>"", \text{LARGE}($B$6:$B$305,$E$1+1-$A$7), "") \]

- **$D$6 formula**
  
  \[ \text{MIN}(C6,$E$2) \]

- **$D$7 formula**
  
  \[ \text{IF} (C7<>"", \text{MIN}(C7,$E$2), "") \]

- **$E$6 formula**
  
  \[ \text{IF} (C6<$E$2, C6, "") \]

- **$E$7 formula**
  
  \[ \text{IF} (C7<$E$2, C7, "") \]

---

Cover formulae
demonstrate censoring by changing censoring time
add a second censoring time in $E$3
modify columns D and E accordingly