Using Thermodynamic Work for Determining Degradation & Acceleration Factors

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Introduction to Damage

• Damage causes aging, we will quantify damage using the concept of thermodynamic work
• Thermodynamics is an energy approach often making it easier to track damage and aging.
Thermodynamics Second Law can be Written in Terms of Damage

- The spontaneous irreversible damage processes that take place in a device interacting with its environment, do so in order to go towards thermodynamic equilibrium with its environment.
Aging Causes Order in the System to Decreases Causing Damage

• Total order of the system plus its environment tends to decrease
• Loss of order equates to increase in entropy
• Not all entropy increase causes damage
• Adding or removing heat, increase or decrease entropy. Yet device damage may not occur
• The spontaneous processes creating disorder are irreversible
• Example: Steal beam will continue to corrode
• The original order created in a manufactured product diminishes in a random manner, and becomes measurable
Entropy Damage

• The entropy generated associated with device damage, is, “entropy damage”

• Entropy Generated $S_{\text{gen}}$ during aging is measured by change
  \[ S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{device}} + \Delta S_{\text{env}} \geq 0 \]
  \[ \text{where for example } \Delta S_{\text{device}} \text{ and } \Delta S_{\text{env}} \text{ is the entropy change to the device and to the environment.} \]

• We can write entropy change in terms of the Non Damage and Damage?
  \[ \Delta S_{\text{device}} = \Delta S_{\text{damage}} + \Delta S_{\text{non-damage}} \geq 0 \]
Complex Systems Entropy Damage

• The total entropy of a system is equal to the sum of the entropies of the parts.

• *This is an important result for thermodynamic damage.* If we can keep tabs on $\Delta S_{\text{Total}}$ over time, we can determine if aging is occurring even in a complex system.

\[
S_{\text{Gen}} = \Delta S_{\text{Total}} = \sum_{i=1}^{N} \Delta S_i = \Delta S_{\text{Sys}} + \Delta S_{\text{Surroundings}} \geq 0
\]

• Alternately, entropy increase equates to free energy decrease

\[\text{Thermodynamic Work} = \phi_i - \phi_f\]
Damage Causes a Loss of Ability to do Useful Work

- Disorder also equates to decrease in free energy
- Loss of the ability to do useful work
- Loss of free energy $F$

\[
\text{Work} = (F_{\text{final}} - F_{\text{initial}})
\]

$F_i$, $F_f$ = Initial, final, free energy
($F_i > F_f$, at constant temperature)
Damage: Entropy Increases or Free Energy Decreases

- Non Equilibrium Thermodynamics → Aging State
  \[
  \frac{dS_{\text{Total}}}{dt} > 0
  \]
  Entropy definition
  
  - Equilibrium Thermodynamics → Non Aging State
  \[
  \frac{dS_{\text{Total}}}{dt} = 0
  \]
  Entropy is as large as possible
  
  \[
  \frac{dF}{dt} < 0
  \]
  Free energy definition
  
  \[
  \frac{dF}{dt} = 0
  \]
  Free energy is as small as possible
Relationship Between Work & Damage

• Define Damage through work
  – When an environment does work on a system, the work that causes aging we call damage and can be quantified as

\[ \text{Damage} = \frac{\text{Partial work}}{\text{Total work needed for failure}} \]
What can we Measure

• The total work is a good measure of the effective damage (as we cannot always measure damage itself)

\[ W_T = W_d + W \]

\[ \text{Damage} = \frac{\sum_{i=1}^{m} W_{d_i}}{W_d} \]

\[ \text{Effective Damage} = \frac{\sum_{i=1}^{m} W_{T_i}}{W_T} \]
Assessing Damage in Non Equilibrium Thermodynamic

• Track the aging path. How does aging occur over time as opposed to assessing a system’s state at end points.
Conjugate Work & Free Energy Approach

• We can sum the work done on the system by the environment

$$\delta W = \sum_a Y_a dX_a \text{ or } W = \int_{X_1}^{X_2} Y \, dX$$
Conjugate Work Variable

- Some common thermodynamic work systems

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<td>Resistor</td>
<td>Voltage (V)</td>
<td>Current (I)</td>
<td></td>
</tr>
</tbody>
</table>
Damage Ratio for Tracking Degradation

- Total work – some work cause damage and some work is due to damage inefficiencies unrelated to system work damage.

\[
\text{Damage} = \frac{\sum_{i=1}^{m} W_{d_i}}{W_d} \quad \text{and} \quad \text{Effective Damage} = \frac{\sum_{i=1}^{m} W_{T_i}}{W_T}
\]

- **Work damage ratio**: work performed to the work needed to cause system failure. In system failure, we exhaust the maximum amount of useful system work.

- All work found must be taken over the same work path.
Damage Ratio (Cont.)

- Cyclic damage can be written over n cycles as
  \[ \text{Damage} = \frac{n}{W_{\text{Failure}}} \sum Y_n \, dX_n \]
  \[ = \frac{\text{Partial cyclic work}}{\text{Total cyclic work to cyclic}} \]

- Non cyclic damage
  \[ \text{Damage} = \frac{\int Y \, dX}{W_{\text{Failure}}} = \frac{\text{Partial work}}{\text{Total work needed for failure}} \]

Same Work Path
Acceleration Factors Using Damage Ratio

- Often the degradation path is separable for time

\[
\begin{align*}
    w &= \int_{t_i}^{t_f} Y(t) \frac{dX(t)}{dt} dt = f(Y, k, E_a) t \\
\end{align*}
\]

- A common case
Acceleration Factors (Cont.)

• Damage between two different environmental stresses \( Y_1 \) and \( Y_2 \), and failure occurs for each at time \( t_1 \) and \( t_2 \). then the damage value of 1 requires

\[
\text{Damage} = \frac{f(Y_2, k, E_a) \tau_2}{f(Y_1, k, E_a) \tau_1} = 1 \quad \text{or} \quad AF(1,2) = \frac{\tau_2}{\tau_1} = \frac{f(Y_1, k, E_a)}{f(Y_2, k, E_a)}
\]

• Then damage can be written

\[
\text{Damage} = \frac{f(Y_1, k, E_a) t_1}{f(Y_2, k, E_a) \tau_2} = \frac{f(Y_1, k, E_a) t_1}{f(Y_2, k, E_a) AF(1,2) \tau_1}
\]
Cyclic Work – How Much Damage/Cycle?

- Cyclic Work cause damage each cycle
- For stress and strain the total cyclic work is

\[ w(\text{cycle}) = \sum_{i=1}^{n} \int_{\text{Area}_i} Sde \]

- The damage is then

\[ \text{Damage} = \frac{\sum_{i=1}^{n} \int_{\text{Area}_i} Sde}{W_{\text{failure}}} = \frac{\text{Sum work per cycle}}{\text{Total work needed for failure}} \]
Fatigue Damage Using Miner’s Rule

• Miner’s rule is a popular approximation for finding damage

• Work $W$ is a function of the cyclic area. For stress $S$, cycles $n$
  \[ W_n = W(S, n) \]

• Miner assumed \( W_n = n W(S) \), therefore

\[
\text{Damage} = \frac{n_1 W(S_1)}{W_{\text{Failure}}} + \frac{n_2 W(S_2)}{W_{\text{Failure}}} + \ldots = \frac{n_1 W(S_1)}{W_{\text{Failure}}} + \frac{n_2 W(S_2)}{W_{\text{Failure}}} + \ldots
\]

\[
W_{\text{Failure}} = N_1 W(S_1) = N_2 W(S_2) = N_3 W(S_3) = \ldots
\]

\[
\text{Damage} = \frac{n_1}{N_1} W(S_1) + \frac{n_2}{N_2} W(S_2) + \frac{n_3}{N_3} W(S_3) + \ldots = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots = \sum \frac{n_i}{N_i}
\]
Example: Miner’s Rule

• Then Miner’s rule can be modified as

\[ N_i = AF(1, i) N_1 \]

\[ \text{Effective Damage} \approx \frac{n_1}{N_1} + \frac{n_2}{AF(1,2)N_1} + \frac{n_3}{AF(1,3)N_1} + \cdots = \sum_{i=1}^{k} \frac{n_i}{AF(1,i)N_1} \]
Example Abrasive Wear

Archard’s wear equation

\[ w = \int F \, dx = C \int_{x_1}^{x_2} P_W \frac{dD}{dt} \, dt = \left[ \frac{kP_W \nu t}{HA} \right]_{x_1}^{x_2} \]

\[ \text{Damage} = \frac{w_2}{W_1} = \left[ \left( \frac{k_P}{HA} \right) \frac{P_{w_2} \nu_2 t}{A_2} \right]_{x_1}^{x_2} = \left[ \left( \frac{k_P}{HA} \right) \frac{P_{w_1} \nu_1 \tau_1}{A_1} \right]_{x_1}^{x_2} \]

Cum Damage for different stresses

\[ \text{Damage} = \sum_i \left( \frac{t_i}{\tau_i} \right) = \sum_i \left( \frac{t_i}{AF(1,i) \tau_1} \right) \]

\[ AF(2,1) = \left( \frac{\tau_2}{\tau_1} \right) = \frac{A_2}{A_1} \frac{\nu_1}{\nu_2} \left( \frac{P_1}{P_2} \right) \]

\( V = \) removed volume of the softer material, \( P = \) normal load (lbs), \( L = \) sliding distance (feet), \( H = \) hardness of the softer material - psi, \( A = \) area, \( D = \) the wear depth removed. Then \( L = \nu t \) for two sliding surfaces rubbing against each other at a constant velocity \( \nu \), \( \tau \) failure time.
Example Creep

$\sigma$ is mechanical stress, $\varepsilon$ strain

$$w = \int_0^{\Delta L} \sigma_w d\varepsilon = \int_0^{\Delta L} \sigma_w \frac{d\varepsilon}{dt} dt = \left[ A \sigma^{b+1} t^Y \right]_0^{\Delta L} = \left[ A(T) \bar{\sigma}^M t^p \right]_0^{\Delta L}$$

**Damage**

$$\text{Damage} = \left[ e^{-Q/K_B \left( \frac{1}{T_2} - \frac{1}{T_1} \right)} \left( \frac{\sigma_2}{\sigma_1} \right)^M \left( \frac{t_2}{\tau_1} \right)^p \right]_0^{\Delta L}$$

$$\text{Damage} = \sum_i \left( \frac{t_i}{\tau_i} \right)^p = \sum_i \left( \frac{t_i}{AF(1,i) \tau_1} \right)^p$$

Cum Damage for different stresses

$$AF_{\text{Creep}} = \left( \frac{\tau_2}{\tau_1} \right) = \left[ e^{-E_a/K_B \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \left( \frac{\sigma_1}{\sigma_2} \right)^{-K}} \right]_0^{\Delta L}$$

A($T$) is a material strength constant, $p$ is the time $t$ exponent where $0<p<1$ for primary and secondary creep stages, $M$ (where $0<(1/M)<1$) is stress $\sigma$ hardening exponent dependent on the material type. Note for secondary creep phase where $p=1$. 

Empirical creep equation,
Example: Thermal Cyclic Fatigue

- \( \Delta T = \text{Temp Change} \), \( n = \text{number of cycles} \), \( N = \text{Cycles to Failure} \), \( A(T) \) Arrhenius Eq.

\[
\begin{align*}
\omega &= \int_{\Delta L} \sigma \, d\varepsilon = \int_{\Delta L} \Delta T \, \frac{d\varepsilon}{dn} \, dn = \left[ A \Delta T^{b+1} n^Y \right]^{\Delta L} = \left[ A(T) \Delta T^M n^p \right]^{\Delta L} \\
Damage &= \left[ e^{-\frac{E_a}{K_B} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)} \left( \frac{\Delta T_2}{\Delta T_1} \right)^M \left( \frac{n_2}{N_1} \right)^p \right]^{\Delta L} \\
Cum \, Damage \, for \, different \, stresses: \\
Damage &= \sum_i \left( \frac{n_i}{N_i} \right)^p = \sum_i \left( \frac{n_i}{AF_D(1, i) N_1} \right)^p \\
AF_{Cyclic \, Creep} &= \left( \frac{N_2}{N_1} \right) = \left[ e^{-\frac{E_a}{K_B} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)} \left( \frac{\Delta T_1}{\Delta T_2} \right)^{-K} \right]^{\Delta L}
\end{align*}
\]

In thermal cycling, a temperature change \( \Delta T \) in the environment, from one extreme to another, causes expansion and/or contraction (i.e. strain) in a material system. The plastic strain (\( \varepsilon \)) caused by the thermal cyclic stress (\( \sigma \)) in the material, over \( n \) cycles. we substitute for non linear stress \( \sigma^{M \sim \Delta T^i} \)
Ex: Mechanical Cyclic Fatigue

- $\Delta T =$ Temp Change, $n =$ number of cycles, $N =$ Cycles to Failure, $A(T)$ Arrhenius Expression with constant

$$w = \int_{\Delta L} \sigma \, d\varepsilon = \int_{\Delta L} G \frac{d\varepsilon}{dn} \, dn = \left[ AG^{j+1} n^p \right]^\Delta L_0 = \left[ G^Y n^p \right]^\Delta L_0$$

$Damage = \left[ \left( \frac{n_2}{N_1} \right)^p \left( \frac{G_2}{G_1} \right)^Y \right]^\Delta L_0$

Cum Damage for different stresses

$$Damage = \sum_i \left( \frac{n_i}{N_i} \right)^p = \sum_i \left( \frac{n_i}{AF_D(1,i) N_1} \right)^p$$

$$AF_D = \left( \frac{N_1}{N_2} \right) = \left( \frac{G_2}{G_1} \right)^b = \left( \frac{W_{PSD2}}{W_{PSD1}} \right)^{b/2}$$

In sine vibration the stress level is in Gs, equal to the sine acceleration $A$ divided by the gravitational constant $g$. In random vibration, a similar quantity is used termed Grms. Consider first the plastic strain ($\varepsilon$) caused by a sinusoidal vibration level $G$ stress ($\sigma$) in the material over $n$ cycles.

$N_2 = \left( \frac{G_1}{G_2} \right)^b \quad N_1 = C_1 G_b^{-k} = C S^{-b}$

$N = C S^{-b}$

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Selected Results in Non Equilibrium Thermodynamics

- Ex. 7: Miner’s Rule for Secondary Batteries

\[ \text{Effective Damage} \approx \sum_{i=1}^{k} \frac{n_i}{N_i} \]

- Miner’s rule using the n cyclic sum over the Depth of Discharge % (DoD%) \( i_{th} \) level stress for battery life pertaining to a certain failure (permanent) voltage drop (such as 10%) of the initially rated battery voltage and then the effective damage done in \( n_i \) DoDs% can be assessed when \( N_i \) is known for the \( i_{th} \) DoD level.
Conclusion

- Non equilibrium thermodynamic assessment
  - Using conjugate work (an energy approach) to assess damage both cyclic and non cyclic over time, an improved method for determining acceleration factors using a work damage approach was presented
Biography

• Dr. Alec Feinberg is the founder of DfRSoftware. He has a Ph.D. in Physics and is the principal author of the book, Design for Reliability. Alec specializes in teaching accelerated training classes in Reliability, Quality, and Shock and Vibration. He utilizes DfRSoftware, the only complete software tool capable of doing all such analytical tasks. He also provides reliability engineering services in reliability and Shock and Vibration to many industries. He previously worked in the electronics industry for over 35 years at ATT Bell Labs, Tyco Electronics, The Analytical Sciences Company, and Advanced Energy. Alec has presented numerous technical papers and won the 2003 RAMS Alan O. Plait best tutorial award for the topic, “Thermodynamic Reliability Engineering”. Alec is also contributing author to the new book on The Physics of Degradation in Engineering Devices and Materials.

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